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MEASURES OF TEMPORAL PATTERN COMPLEXITY

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ABSTRACT

In this study, three measures of temporal pattern complexity were compared with regard to their perceptual validity. The first measure, based on the work of Tanguiane (1993), uses the idea that a temporal pattern can be described in terms of (elaborations of) more simple patterns, which occur simultaneously at different levels.

The second measure is based on the complexity measure for finite sequences proposed by Lempel and Ziv (1976), which is related to the number of steps in a self-delimiting production process by which such a sequence is presumed to be generated.

The third measure, newly developed here, is rooted in the theoretical framework of rhythm perception of Povel and Essens (1985). It takes into account the ease of coding a temporal pattern and the complexity of the segments resulting from this coding. The perceptual validity of the three measures was evaluated in an experiment in which subjects judged the complexity of 35 temporal patterns.

Correlations between the three measures and the collected complexity judgments indicated that the third measure is a much better predictor of temporal pattern complexity than the other two measures. This is probably due to the fact that this measure, unlike the other two, is based on an empirically tested model of rhythm perception that takes into account the isochronous frame against which the rhythm is perceived. Reasons for the differences in performance between the three measures are discussed.

INTRODUCTION

The notion of complexity has generally been studied in the context of information theory and is closely connected with concepts such as randomness, information, regularity, and coding (Calude, 1994). Classical information theory, as well as notions of randomness, based on Shannon's concept of entropy (Shannon, 1948), relies on a priori knowledge of a probability distribution. In that respect, it does not allow one to speak of a particular object or outcome as being random or complex.

In general, an object's complexity reflects the amount of information embedded in it. The representation of the object's information is achieved via coding. When a human being enters the equation, however, care must be taken in interpreting the notion of complexity, which necessarily becomes subjective. Moreover, depending on the context, only certain types of codes may be perceptually significant and hence coding efficiency or complexity must be considered within such constraints (Chater, 1996). This is well known, for example, in the field of visual perception (Leeuwenberg, 1971). In order to obtain complexity measurements from subjects, Pressing (n.d.) suggests equating complexity with

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difficulty of learning, which in turn could be expressed by recognition or production.

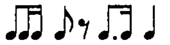
In this work, we consider the complexity of temporal patterns. Our aim is to construct a measure of complexity that corresponds to a high degree with a human's subjective notion of complexity. Pressing (n.d.) discusses three notions of complexity. The first is termed *hierarchical complexity*, which refers to structure on several levels simultaneously. The receiver is then able to perceive structure on one or more levels, inducing an appropriate complexity judgement. A general approach to hierarchical structure in perception has been proposed by Leyton (1986). The divisible nature of Western rhythms lends itself to hierarchical subdivision and reveals regularity of time organization on several levels simultaneously (Lerdahl & Jackendoff, 1983). The second type of complexity is referred to as *dynamic complexity*. This notion refers to the degree of stationarity or change with respect to time, in the sensory input. A highly nonstationary stimulus would tend to be perceived as being complex. In music, rhythms tend to be stationary or periodic in that events or groups of events are repeated in time. Finally, the third type of complexity, called *generative complexity*, refers to a tendency towards the most economical description (Hochberg & McAlister, 1953; Chater, 1996).

In this paper, we examine three new measures of complexity of temporal patterns. The first measure is based on the work of Tanguiane (1993), and uses the idea that a rhythmic pattern can be described in terms of (elaborations of) more simple patterns, which occur simultaneously at different levels. The second measure is based on the complexity measure for finite sequences proposed by Lempel and Ziv (1976), which is related to the number of steps in a self-delimiting production process by which such a sequence is presumed to be generated. Finally, the third measure proposed is rooted in the theoretical framework of rhythm perception discussed in Povel and Essens (1985). This measure takes into account the ease of coding a temporal pattern and the (combined) complexity of the segments resulting from this coding. The measure presupposes the existence of a "temporal grid" or time scale consisting of isochronic intervals, which is selected among a set of possible grids according to the "economy principle" (Povel, 1984). All three measures, which will be discussed in detail below, capture one or more of the types of complexity discussed above.

SELECTION AND NOTATION OF RHYTHMS

Before we proceed to explain the three measures, we must define the domain of rhythms studied. First, we restrict ourselves to quantized rhythms, i.e., rhythms as notated in a score, without timing deviations due to performance. Therefore, these rhythms can be described fully in musical notation. Without loss of generality and for the ensuing discussion, we use the sixteenth note as the smallest note duration (unit of length). Furthermore, the rhythms studied are supposed to repeat or loop infinitely and thus form an infinite sequence of events. Finally, we notate a rhythmic pattern as a string of ones and zeros, in which the symbol '1' represents a note onset and '0' represents no note onset. Of course, the smallest quantization level must be used for the encoding. For example, the pattern

would be represented by 1011100010011000. We should emphasize that in this notation, only inter-note intervals are relevant and so the pattern



is represented by exactly the same string as above. Now, we are ready to give the definitions of the complexity measures.

T-Measure (Tanguiane measure)

The first measure we consider is based on the work of Tanguiane (1993). A basic notion in the theory of Tanguiane is that of *elaboration* (Mont-Reynaud & Goldstein, 1985). Figure 1 gives an example of the elaboration of a quarter note. As can be seen, the quarter note is elaborated into patterns consisting of two notes on the second row, which in turn are further elaborated into patterns of three notes on the third row, which are all finally elaborated into a pattern of four notes on the last row. Thus, all patterns that are linked by a line, either directly

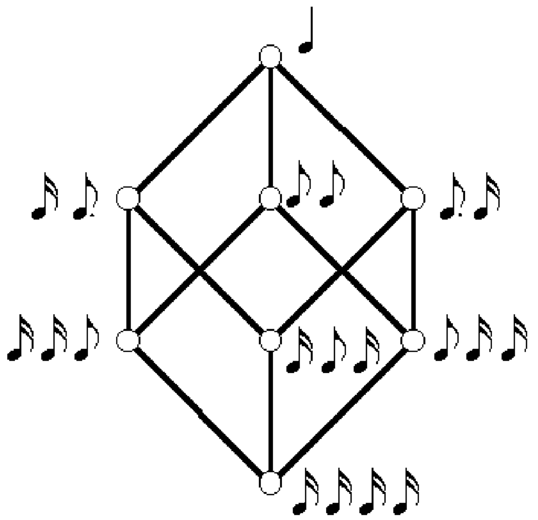


Fig. 1. Elaboration of a quarter note.

or indirectly, to a higher pattern form elaborations of that higher pattern.

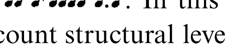
Tanguiane shows how a rhythmic pattern can be described by rhythmic configurations at several levels simultaneously, e.g., at the eighth note level, at the quarter note level, and so on. Of course, for each such level, we use an appropriate partially ordered set similar to the one shown in Figure 1. This representation on several levels accounts for the structural redundancy inherent in divisible rhythms, such as those found in Western music. The T-measure intends to capture hierarchical complexity. As shown for the quarter note example above, not all patterns are elaborations of other patterns. Within any given set of patterns, those patterns that are not elaborations of any other pattern are called *root patterns.*

These root patterns form the core of Tanguiane's notion of complexity, where he states that the root patterns are "... irreducible to each other [and] can be used for estimating the complexity of rhythm" (Tanguiane, 1993). It may be noted that the set of root patterns defines a monotone Boolean function (Yablonsky, 1989, p. 37). Tanguiane defines complexity as the number of root patterns required to generate the given rhythmic pattern on a given structural level. This corresponds to the number of minimal true vectors of the corresponding monotone Boolean function. Finally, the overall complexity of the rhythmic pattern is defined by taking the maximum of all complexities over all the structural levels. It is well known (Gilbert, 1954) that the maximum number of minimal true vectors, of a monotone Boolean function of n variables is equal to

$$M_n = \binom{n}{\lfloor n/2 \rfloor}$$

and hence determines the maximum possible number of root patterns and consequently the maximum complexity under the T-measure. For the elaborations of the quarter note, as shown in Figure 1, the number of root patterns can be no larger than 3. Therefore, for this example, the T-Measure of complexity is an integer between 1 and 3.

Still, such a definition of complexity is problematic since some structural levels may not lend themselves to subdivision. The reason is that the occurrence of certain durations in the rhythmic pattern may prevent subdivision into some other patterns. For example, the presence of a dotted eighth note, which is essentially comprised of 3 sixteenth notes, prohibits the subdivision of the entire pattern into eighth notes. However, the definition of complexity can be slightly modified by simply taking the maximum over all *allowable* structural levels, that is, over those levels which can be subdivided into the duration of interest. Furthermore, it can be decided a priori, if desired, to restrict oneself to only certain structural levels. For instance, we may wish to ignore subdivisions into patterns equal to the duration of 5 sixteenth notes, even though such subdivisions may be possible since the rhythm is assumed to be infinite. Let us now look at a specific example illustrating the application of the T-Measure.

Example 1: Consider the pattern 111011111001 which can be written as . In this example, we only take into account structural levels generated by dotted eighth notes and quarter notes. Remember that the temporal sequence is assumed to be infinite. On the dotted eighth level, we find the following sub-patterns: 100, 111, and 101. Note that in this example, 110 cannot be a pattern (even though it is found in the sequence) because the immediately following pattern always needs to start with a 1, representing a tone onset. Clearly, 111 and 101 are both elaborations of 100, which is the only root pattern on that level, implying a complexity of 1. On the quarter note level, we find: 1110, 1111, and 1001. In this case, we see that 1110 and 1110 are both root patterns, 1111 being the elaboration of both of them and hence the complexity on that level is equal to 2. Taking the maximum over all considered levels, we find the overall complexity to be equal to 2.

LZ-Measure (Lempel-Ziv measure)

Another approach for quantifying complexity of rhythms is to use the popular measure proposed by Lempel & Ziv (1976). This complexity measure captures the number of "new" substrings discovered as the sequence evolves from left to right (as is the case in music). As soon as a new substring is found, the complexity increases by 1. The measure essentially takes into account repetitions of patterns on all structural levels, thus capturing both dynamic as well as hierarchical complexities. It can easily be shown that the Lempel-Ziv (LZ) complexity of a periodic binary sequence is finite. This is, of course, desirable for quantifying complexity of temporal patterns. It should also be pointed out that the LZ complexity in general is not well suited for very short sequences and thus the assumption of cyclical rhythms is very useful. The measure is intended to capture the multi-level redundancy embedded in the rhythmic pattern without regard to any perceptual mechanisms involved in coding it. Thus, the measure does not take into account the possibility that some of the information embedded in the sequence may not be perceptually relevant to a human listener. Therefore, it can be used as a reference point for other measures that do incorporate perceptual constraints as they should exhibit greater correspondence to subjective judgements of complexity than the LZ-Measure. Let us now look at the same example as above and compute the LZ complexity for that rhythm.

Example 2: We shall write the sequence representing the rhythm two times, since it will turn out that the LZ complexity needs the second instance of the sequence to reach its final value. So, the sequence is:

111011111001 111011111001

In the sequence below, new substrings are delimited by dots.

1•110•1111•100•1 1110•11111001...

Note that after the last dot, no new substrings exist. The LZ complexity for this sequence is equal to 5, which is equal to the number of dots.

PS-Measure (Povel-Shmulevich measure)

The PS-Measure is rooted in the theoretical framework of perception of temporal patterns discussed in Povel and Essens (1985). A basic notion of that model is that a listener attempts to establish an internal clock (beat) that segments the rhythm into equal intervals. While there is no physiological explanation of the internal clock nor reason for its existence, structural regularity of auditory stimuli induce in the listener a certain response, be it tapping of a foot or merely recognition of the rhythmic organization. Presumably, this temporal segmentation serves to reduce the coding complexity of the stimulus, which would be consistent with the Gestalt simplicity principle, implying that sensory input is coded in the simplest possible way (Chater, 1996). This aspect of the model is concerned with generative complexity.

The induction of the clock is determined by the distribution of accents in the sequence (see also Parncutt, 1994; Jones & Pfordresher, 1997). As several possible clocks will fit with any given rhythm, it is assumed that the clock which best fits the distribution of accents in the rhythm is the one actually induced. This clock is referred to as the best clock. Furthermore, the ease with which the best clock is induced depends on how well it fits the distribution of accents. After the selection of the best clock, the rhythm is represented by coding the segments produced by this clock.

Discussing the complexity of rhythms, the authors state that a "... given temporal pattern will be ... judged complex when either no internal clock is induced or, where it is induced, when the coding of the pattern is relatively complex" (Povel & Essens, 1985). In light of that, the proposed measure of complexity should be a combination of the induction strength of the best clock on the one hand and the efficiency of coding the rhythm on the other.

The first part of the PS-Measure thus pertains to the induction strength of the best clock, which is captured by the C-score (Povel and Essens, 1985). The C-score is computed by taking into account a weighted combination of the number of clock ticks that coincide with unaccented events and with silence:

$$C = W \cdot s + u$$

in which s stands for the number of clock ticks coinciding with silence and u for the number of unaccented events. The lower the score, the higher the induction strength of the clock; hence higher scores correspond to higher complexity.

The second part of the PS-measure pertains to the efficiency of the code. In determining coding complexity, we distinguish between four types of possible segments as shown in Figure 2: an empty segment (E), an equally subdivided segment (S_k, where k indicates the number of equal subdivisions), an unequally subdivided segment (U), and finally a segment which begins with silence (N).

To compute the coding complexity, a different weight is associated with each type of segment. Weights $d_1, ..., d_4$ correspond respectively to the four types of segments distinguished above. Finally, a weight d_5 is used in order to account for repetitions of segments. Specifically, if a segment is different from the segment following it, d_5 is added to the sum of all weights accumulated so far. The rationale behind this is that two different consecutive segments are likely to increase complexity.

Now, the formula for the total coding complexity is:

$$D = \sum_{i=1}^{n} c_i + m \cdot d_5$$

where $c_i \in \{d_1, ..., d_4\}$ is the weight of the i^{th} segment, n is the number of segments, and m is the number of consecutive segment pairs containing different segments.

<table>
<tr><td>|... |</td><td>Empty (E)</td></tr>
<tr><td>|. |. |</td><td>Equally subdivided (S_2)</td></tr>
<tr><td>|.. | |</td><td>Unequally subdivided (U)</td></tr>
<tr><td>. |.. |</td><td>Starting with silence (N)</td></tr>
</table>

Fig. 2. Four types of segments.

Finally, the PS-Measure is defined as the weighted combination of the induction strength of the clock and the total coding complexity:

$$P = \lambda \cdot C + (1 - \lambda) \cdot D$$

where C is the induction strength of the best clock and D is the total coding complexity obtained by segmenting the rhythm with that clock.

Two parameters which must be determined are W and λ. W is the weight used in the formula to compute C (see above). The parameter λ represents the relative importance of clock induction strength and coding efficiency. Before discussing the procedure for determining all the necessary parameters, let us first look at an example.

Example 3: Consider the rhythm pattern represented by

1010 1010 1110 1100

which can be written as .

Suppose that the best clock for this rhythm has unit 4 and location 1 and a value of $C = 1$. Suppose that $\lambda = 0.5$. The codes of the four segments are respectively: S_2, S_2, 1-1-2, and 1-3. Suppose further that $d_2 = 1$, $d_3 = 3$, and $d_5 = 1$. Since the second segment is different from the third which is in turn different from the fourth, $m = 2$, and thus, the overall coding complexity is equal to $D = 1 + 1 + 3 + 3 + 2 \cdot (1) = 10$. Finally, $P = 0.5 \cdot 1 + 0.5 \cdot 10 = 5.5$, which is the overall complexity of the rhythm pattern.

PARAMETER ESTIMATION

The parameters were determined by utilizing the results of an experiment reported in Essens (1995). In Experiment 3 of that work, twenty subjects were asked to make complexity judgements on 24 rhythmic patterns, on a scale of 1 to 5. All parameters were optimized so as to increase the correlation between the average judged complexity collected in that experiment and the PS-Measure. More formally, let $\vec{\theta} = (\lambda, W, d_1, ..., d_5)$ be the vector of parameters used in the PS-Measure and let $\vec{C}_J$ and $\vec{C}_{PS}$ be 24-length vectors containing complexities from human listeners (averaged) and the PS-Measure respectively. Then, the goal is to find the parameter $\hat{\theta}$ so that

$$\hat{\theta} = \arg \max_{\vec{\theta}} \, [\rho(\vec{C}_J, \vec{C}_{PS})]$$

where $\rho(\cdot, \cdot)$ is the correlation.

To achieve this, simplex search as well as quasi-Newton search methods were used. The parameters maximizing the correlation are shown in Table 1. The resulting correlation between the average judged complexities and the PS-Measure complexities computed with these parameters is $\rho\left(\vec{C}_J, \vec{C}_{PS}\right) = 0.83$.

As shown by Essens (1995), the C-score had no significant effect in predicting complexity. This is supported by the fact that λ, the weight given to C, is low, implying that the coding complexity plays a much more important role[1]. As expected, empty (E) segments contribute less to complexity than equally and unequally subdivided segments, as is shown by the value of d_1. What is surprising, however, is that the weight associated with equally subdivided segments (d_2) is not significantly lower than the weight associated with unequally subdivided segments (d_3). The weight associated with segments that do not begin with a note onset is close to zero. This condition occasionally occurs because the optimal value of W is different from the one used by Essens (W = 4), consequently changing the best clock. Finally, the weight d_5 indicates that *variations* between segments do play a role in determining complexity, which again supports the findings of Essens (1995).

EXPERIMENT

The purpose of this experiment was to determine how well the PS-Measure predicts the judged complexity of a set of temporal patterns not used in the computation of the measure. Also the predictive powers of the T-measure and the LZ-measure were determined. For that purpose subjects judged a set of 35 temporal patterns as described below.

Table 1. Estimated parameter values used in the PS measure.

λ	W	d_1	d_2	d_3	d_4	d_5
0.2223	1.1695	0.0235	1.2722	1.2955	0.0736	0.7931

Method

Participants. Twenty-five subjects, graduate, undergraduate students, and faculty at the University of Nijmegen, participated in the experiment. The median age of the subjects was 20 years. All of them were musicians, with an average of 9.2 years of practical musical experience.

Stimuli. The stimuli used in the experiment consisted of rhythmic patterns, each containing four different intervals, namely 200 ms, 400 ms, 600 ms, and 800 ms (which may respectively be notated as 1, 2, 3, 4). The patterns were all permutations of the combination 1 1 1 1 1 2 2 3 4. The 800 ms interval was always at the end of a pattern. The 35 patterns used in the experiment are displayed in Table 2.

Stimulus presentation. The patterns were generated on a Rhodes model 760 MIDI-synthesizer, using the middle C (261,6 Hz) of the Marimba sound, which was emitted via a Kawai KM-20 active speaker. There were no differences in intensity or pitch between the tones. Stimulus presentation was controlled by a Power Macintosh 8200/120. Each pattern was repeated three times, so one entire stimulus had a length of four patterns. As each pattern lasted for 3.2 seconds, an entire stimulus had a duration of 3.2 seconds * 4 (repeats) = 12.8 seconds. Patterns were presented in random order, different for each subject.

Procedure. Subjects were seated in front of the computer screen. After the presentation of each stimulus, subjects were required to judge the complexity of the stimulus on a 5-point scale (1 = simple, 5 = complex), displayed on the screen. Subjects were asked to imagine how difficult it would be to reproduce the rhythms. Subjects could listen to a sequence more than once, and change their judgment before proceeding to the next stimulus. Each subject first trained with five practice trials to get acquainted with the procedure. The entire experiment lasted approximately twenty minutes.

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[1] It should be noted that the C-score did play a role in a rhythm reproduction task reported by Essens (1995).

Table 2. The 35 patterns used in the experiment, together with their mean judged complexity. Each vertical line represents a tone. The smallest interval between two tone onsets is 200 ms. The dots have no physical meaning whatsoever; their function is to represent the relative duration of the intervals.

<table>
<tr><th>No.</th><th>Pattern</th><th>Comp.</th><th>No.</th><th>Pattern</th><th>Comp.</th></tr>
<tr><td>1</td><td>|||||..||.|.|...</td><td>1.56</td><td>19</td><td>||..||.||||.|...</td><td>2.64</td></tr>
<tr><td>2</td><td>|||.|.|||..||...</td><td>2.12</td><td>20</td><td>||..||.|||.||...</td><td>3.24</td></tr>
<tr><td>3</td><td>|.|||.|||..||...</td><td>2.08</td><td>21</td><td>|||||.||.|..|...</td><td>3.08</td></tr>
<tr><td>4</td><td>|.|.|||||..||...</td><td>1.88</td><td>22</td><td>||||.|..|||.|...</td><td>3.04</td></tr>
<tr><td>5</td><td>|..||.|.|||||...</td><td>1.80</td><td>23</td><td>|||..||.|||.|...</td><td>3.04</td></tr>
<tr><td>6</td><td>|||.|||.||..|...</td><td>2.44</td><td>24</td><td>|.|||..|.||||...</td><td>2.56</td></tr>
<tr><td>7</td><td>|.||||.||..||...</td><td>2.20</td><td>25</td><td>|.|..||||.|||...</td><td>2.56</td></tr>
<tr><td>8</td><td>||..|||||.|.|...</td><td>2.56</td><td>26</td><td>||||.|.|..|||...</td><td>2.84</td></tr>
<tr><td>9</td><td>||..|.|||.|||...</td><td>3.00</td><td>27</td><td>||.|||.|..|||...</td><td>3.60</td></tr>
<tr><td>10</td><td>|.|||.||||..|...</td><td>2.04</td><td>28</td><td>||.|..|||.|||...</td><td>2.68</td></tr>
<tr><td>11</td><td>|||.||..||.||...</td><td>2.76</td><td>29</td><td>|.||||.|..|||...</td><td>3.28</td></tr>
<tr><td>12</td><td>||.||||.|..||...</td><td>2.72</td><td>30</td><td>|..|||||.||.|...</td><td>3.08</td></tr>
<tr><td>13</td><td>||.||.||||..|...</td><td>3.00</td><td>31</td><td>||||.|||..|.|...</td><td>3.52</td></tr>
<tr><td>14</td><td>||..||.||.|||...</td><td>3.16</td><td>32</td><td>||||..||.||.|...</td><td>3.60</td></tr>
<tr><td>15</td><td>|..|||.|||.||...</td><td>2.04</td><td>33</td><td>||.||||..||.|...</td><td>3.04</td></tr>
<tr><td>16</td><td>||.||||.||..|...</td><td>2.88</td><td>34</td><td>||.|..|||||.|...</td><td>2.88</td></tr>
<tr><td>17</td><td>||.|||.|||..|...</td><td>2.60</td><td>35</td><td>|.|..|||.||||...</td><td>3.08</td></tr>
<tr><td>18</td><td>||.|||..||.||...</td><td>2.60</td><td></td><td></td><td></td></tr>
</table>

RESULTS

The averages of the scores for each item were computed over all subjects and are displayed next to each stimulus in Table 2. To assess the reliability of the average of the responses, we computed the Cronbach alpha measure. This measure provides us with a lower bound for the correlation between the average scores of the performed experiment with another identical experiment containing the same number of subjects. For our experiment, α = 0.88, which indicates a high degree of internal consistency.

The correlations between the average judgements and the values as predicted by the complexity measures discussed herein were computed. The complexities given by the PS-Measure were computed by using the estimated parameters given in Table 1. The correlations between the judged complexities and the complexities predicted by the T-Measure, LZ-Measure, and the PS-Measure are: $r = 0.02$, $r = 0.15$, and $r = 0.75$ respectively.

DISCUSSION

Essens (1995) showed that the C-score had no significant effect in predicting complexity, although it did play a role in a rhythm reproduction task. Because of the nature of the instructions given to subjects, linking complexity with ease of reproduction, we expected the C-score to contribute at least partially to complexity judgements. This indeed turned out to be the case insofar as the mean complexity judgements follow a general trend upwards with respect to the ordering of the stimuli, which are arranged by increasing values of their C-scores. This is illustrated in Figure 3. However, we hypothesized that the coding complexity of temporal patterns played a more significant role in determining the overall complexity judgements. This was confirmed, in part, during the so-called "training phase" of the PS-Measure, which assigned a weight of approximately 80% to the coding complexity.

The correlation between the average responses for each item in the above experiment and the values as predicted by the PS-Measure with the parameters given in Table 1, was computed and found to be $r = 0.75$. The value obtained indicates that the PS-Measure is a robust measure of complexity of temporal patterns in the sense that the parameters estimated from one set of patterns yield high predictive power for another set. Furthermore, it could be inferred that this measure captures the essential components in rhythm that directly con-

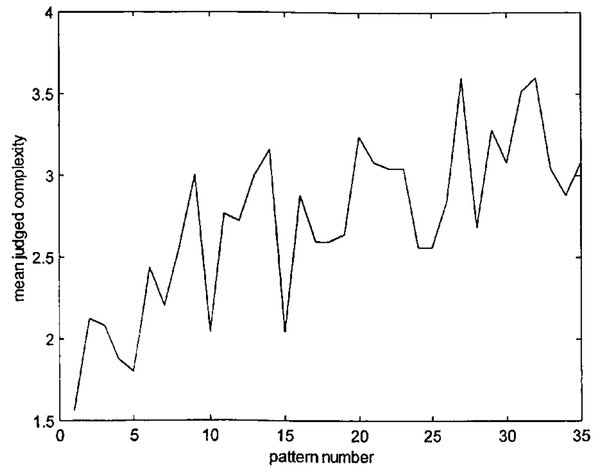

Fig. 3.

tribute to the perception of complexity. The training phase of the measure, which consisted of finding the optimal parameters corresponding to various features of the rhythm, succeeded in determining the relative importance of these components.

It should also be mentioned at this time that the T-Measure and the LZ-Measure correlated with complexity judgements from the Essens (1995) experiment with values of $r = 0.13$ and $r = 0.12$ respectively. The correlations with the judgements from the experiment discussed here are $r = 0.02$ and $r = 0.15$ for the T-Measure and the LZ-Measure, respectively. The low correlation values imply that these two measures are unlikely to be perceptually reliable measures of complexity of temporal patterns.

The T-Measure's poor performance is most likely due to the lack of perceptual validity supporting its use. This measure is essentially based on the assumption that root patterns are the major contributing factors of complexity of temporal patterns. We are not aware of any studies supporting this assumption. Moreover, the T-Measure does not possess enough resolution to be able to accurately predict complexity judgements. For example, as shown above, the elaborations of a quarter note would only permit the complexity to be 1, 2, or 3.

The LZ-Measure is likely to perform much better if applied to longer rhythms, since the LZ complexity is not well suited for short sequences, even when they are assumed to be cyclical. After all, there is very little information embedded in a short sequence and the assumption of it being cyclical does not add any new information. It would thus be appropriate to consider the LZ-Measure for use with much longer rhythms.

It is not surprising that the PS-Measure outperformed the T-Measure as well as the LZ-Measure, because the PS-measure incorporates perceptual information and is based on an empirically tested model of rhythm perception.

While the PS-Measure has not been tested on temporal patterns with timing deviations due to performance, such as those found in real music and which undoubtedly add another dimension to complexity, it nevertheless seems to be able to capture the basic structural components of musical rhythm that contribute to the perception of complexity. We would like to emphasize, however, that the factors that we expected to contribute to the subjects' notions of complexity are very much a function of their cultural background. As is implicitly assumed in the T-Measure and the PS-Measure, the temporal patterns we consider reflect Western divisible rhythms. It is to be expected that a listener used to additive rhythms with complex duration ratios will have significantly different judgements of complexity for the same set of stimuli than a Western listener.

In Shmulevich and Povel (1998), it is argued that a successful and perceptually salient measure of rhythm complexity can be used in a music pattern recognition system (Coyle & Shmulevich, 1998, Shmulevich et al., 1999) by allowing it to determine relative weights of pitch and rhythm errors. The PS-Measure seems to be an appropriate starting point for further development in this direction.

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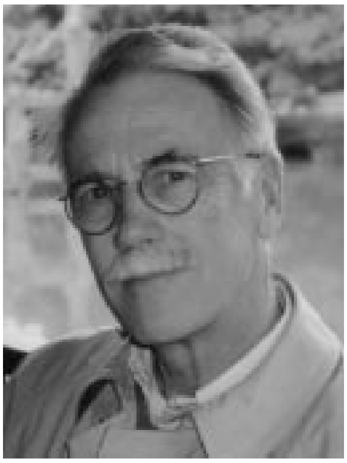

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